Production and Labor Networks

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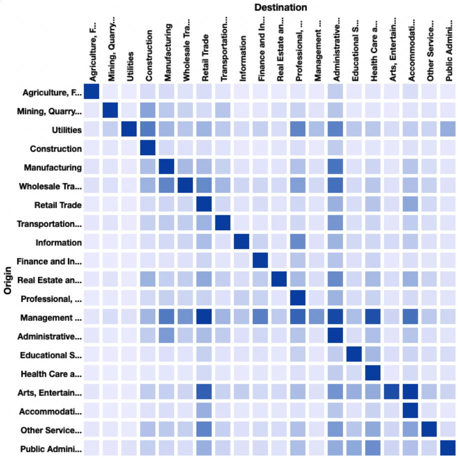
Introduction

- Fast growing literature on role of production networks across sectors to account for the role of idiosyncratic shocks on aggregate outcomes
  - Acemoglu et al. (2012), Baqaee & Farhi (2019, 2020), Bigio & La'O (2020), Liu (2021)
- Other networks in the economy are also important!
  - Acemoglu et al. (2015): Financial networks
  - Winberry & von Lehm (2021): Investment networks
  - Lim (2018), Taschereau-Dumouchel (2022), Cardoza et al (2022): Firm–firm trade
- **This paper**: Study the role of the *labor market* network.
  - How does the production economy react when we take into account costly and slow reallocation of factors of production?
  - Pilossoph (2012): $\simeq 10\%$ of unemployment volatility due to intersectoral frictions

J2J Transitions



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Introduction

- The *labor market* network
  - Features a lot of within–sector transitions
  - Labor as a *durable* good
  - Involves worker decision!
- How do production and labor networks interact?

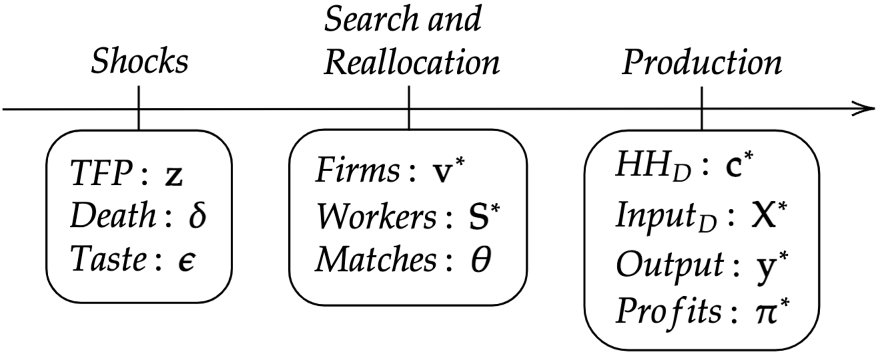
What I do

- Introduce production network structure in a multi-sector labor search model
- Key inputs
  - $\kappa, p(\theta), q(\theta)$: Role of search frictions
  - Ω: Input–Output matrix
  - Φ: Worker's switching cost matrix
- Characterize the solution in a stationary environment
- Study the economy's reaction to shocks under different structures
- Standard models are nested within this framework

Environment

- Infinite–horizon, discrete–time
- n sectors with inter–sectoral input–output (I–O) linkages
- **Firms**: On each sector i, a continuum of single–worker firms
  1. Search for a worker to produce (if vacant)
  2. Source inputs from all sectors (if filled vacancy)
- **Workers**: Every period choose in which sector to work. Intersectoral J2J transitions are costly.
- All workers act as a family that consume an aggregate good (numéraire)

Timing


Shocks
Search and Reallocation
Production
TFP : z
Death : δ
Taste : ϵ
Firms : v*
Workers : S*
Matches : θ
HH_D : c*
Input_D : X*
Output : y*
Profits : π*

1. Firms – Search block

- There's a labor market on each sector i
  - Matching function: $M_i(u, v)$
  - Market tightness: $\theta_i \equiv v_i / u_i$
  - Matching probabilities: $q(\theta_i), p(\theta_i) = \theta_i q(\theta_i)$
- Firms pay κ_i to post a vacancy and die with probability δ_i
- Firm's value functions

$$J_i(\mathbf{z}) = \pi_i(\mathbf{z}) - w_i(\mathbf{z}) + \beta \mathbb{E}_{\mathbf{z}}\left[\left(1 - \delta_i^F(\mathbf{z}')\right) J_i(\mathbf{z}') + \delta_i^F(\mathbf{z}') V_i(\mathbf{z}')\right]$$
$$V_i(\mathbf{z}) = -\kappa_i + \beta \mathbb{E}_{\mathbf{z}}\left[q(\theta_i(\mathbf{z}')) J_i(\mathbf{z}') + (1 - q(\theta_i(\mathbf{z}'))) V_i(\mathbf{z}')\right]$$

- Measure of entrant/poster firms is pinned down by free entry $V_i = 0$
- $\delta_i^F(\mathbf{z})$ is endogenous (takes into account JD from reallocation)

1. Firms – Search block

- Firm's value functions

$$J_i(\mathbf{z}) = \pi_i(\mathbf{z}) - w_i(\mathbf{z}) + \beta \mathbb{E}_{\mathbf{z}}\left[\left(1 - \delta_i^F(\mathbf{z}')\right) J_i(\mathbf{z}') + \delta_i^F(\mathbf{z}') V_i(\mathbf{z}')\right]$$
$$V_i(\mathbf{z}) = -\kappa_i + \beta \mathbb{E}_{\mathbf{z}}\left[q\left(\theta_i(\mathbf{z}')\right) J_i(\mathbf{z}') + \left(1 - q\left(\theta_i(\mathbf{z}')\right)\right) V_i(\mathbf{z}')\right]$$

- $\pi_i \longrightarrow$ Value added (revenue net of intermediate input expenses)
- $\delta_i^F \longrightarrow$ Job destruction probability accounts for intersectoral mobility
- $w_i \longrightarrow$ Nash Bargaining

1. Firms – Technology

- Firms in sector i produce according to Cobb–Douglas technology

$$y_i = z_i^{\frac{1}{1-\eta_i}} \xi_i \left[\prod_{j=1}^{n} x_{ij}^{\omega_{ij}} \right]^{\eta_i}$$

- z_i is sector–specific TFP
- ξ_i is normalizing constant
- x_{ij} denotes $i's$ demand for $j's$ output
- ω_{ij} are input shares
- η_i is degree of DRS

1. Firms – Input choice

- Firm's input choice is a **static** problem

$$\pi_i(\mathbf{z}) \equiv \max_{\{x_{ij}\}_{j=1}^{n}} \left\{ p_i y_i(\mathbf{x}_i,) - \sum_{j=1}^{n} p_j x_{ij} \right\}$$

- Output per worker

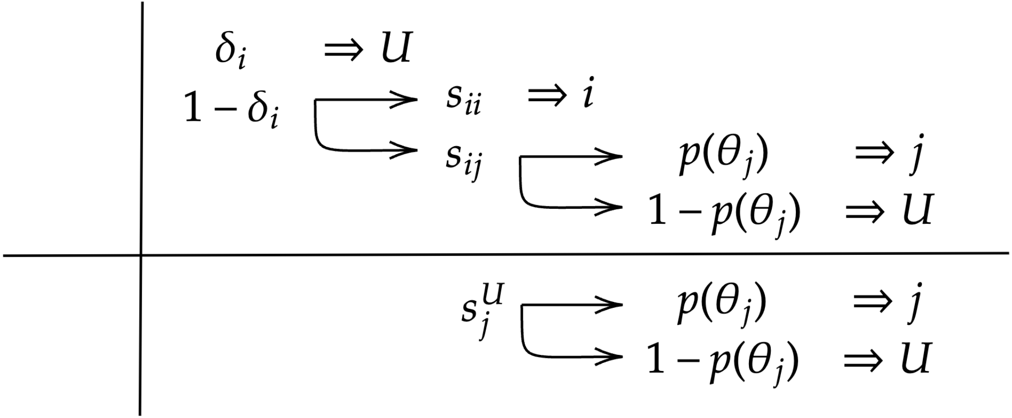
$$y_i^*(\mathbf{z}) \equiv z_i \left[\prod_{j=1}^{n} (p_i/p_j)^{\omega_{ij}} \right]^{\gamma_i}, \quad \gamma_i \equiv \frac{\eta_i}{1-\eta_i}$$

- Sector value added

$$\pi_i(\mathbf{z}) = (1-\eta_i)\, p_i y_i^*(\mathbf{z})$$

- Profits depend on **z** since TFP shapes prices and thus firm's cost structure

Timing for Workers



2. Workers – Value functions

- Workers can choose to move across sectors

$$
\begin{aligned}
W_i(\mathbf{z}) &= w_i + \beta\delta_i \mathbb{E}_{\mathbf{z}}\left[U(\mathbf{z}')\right] \\
&\quad + \beta(1-\delta_i)\mathbb{E}_{\mathbf{z}}\left[\mathbb{E}_{\epsilon}\left\{\max\left\{W_i(\mathbf{z}') + \epsilon_i, \max_{j\neq i}\left(\mathcal{W}_j(\mathbf{z}') - \varphi_{ij}^{E} + \epsilon_j\right)\right\}\right\}\right] \\
U(\mathbf{z}) &= b + \beta\mathbb{E}_{\mathbf{z}}\left[\mathbb{E}\left\{\max_{i\in\mathcal{I}}\left(\mathcal{W}_i(\mathbf{z}') - \varphi_j^{U} + \epsilon_i\right)\right\}\right] \\
\mathcal{W}_i(\mathbf{z}) &= p(\theta_i)\cdot W_i(\mathbf{z}) + (1 - p(\theta_i))\cdot U(\mathbf{z})
\end{aligned}
$$

- Discrete choice problem, where $\mathcal{W}_j$ is expected value of searching in sector j
- $\epsilon_i \sim GEV_I$ are *preference* shocks
- φ describe the switching costs across sectors

2. Workers – Choosing a sector

- Expected value of seaching in sector j

$$\mathcal{W}_j(\mathbf{z}) = p(\theta_j) \cdot W_j(\mathbf{z}) + (1 - p(\theta_j)) \cdot U(\mathbf{z})$$

- Well–known result in discrete choice literature: $\epsilon_i \sim GEV_I$ yield probabilities

$$s_{ij}^E(\mathbf{z}) = \Pr(i \rightarrow j) = \begin{cases} \frac{e^{\mathcal{W}_j(\mathbf{z}) - \varphi_{ij}^E}}{e^{W_i(\mathbf{z})} + \sum_{k \neq i}^n e^{\mathcal{W}_k(\mathbf{z}) - \varphi_{ik}^E}} & i \neq j \\ \frac{e^{W_i(\mathbf{z})}}{e^{W_i(\mathbf{z})} + \sum_{k \neq i}^n e^{\mathcal{W}_k(\mathbf{z}) - \varphi_{ik}^E}} & i = j \end{cases}$$

$$s_j^U(\mathbf{z}) = \Pr(U \rightarrow j) = \frac{e^{\mathcal{W}_j(\mathbf{z}) - \varphi_j^U}}{\sum_{k=1}^n e^{\mathcal{W}_k(\mathbf{z}) - \varphi_k^U}}$$

- Relative values of being employed at each sector determines $\Pr(move)$

2. Workers – Choosing a sector

- Transition probabilities from unemployment can be expressed as

$$s_j \cdot \frac{\exp\left(-U\right)}{\exp\left(-U\right)} = \frac{\exp\left[p\left(\theta_j\right)\left(W_i - U\right)\right]}{\sum_k \exp\left[p\left(\theta_k\right)\left(W_k - U\right)\right]}$$
$$= \frac{\exp\left[p\left(\theta_j\right)\alpha_j S_j\right]}{\sum_k \exp\left[p\left(\theta_k\right)\alpha_k S_k\right]}$$

  using the Nash bargaining solution: $W_i - U = \alpha_i S_i$.

- Same reasoning for s^E_{ij} leads to

$$s^E_{ij} = \begin{cases} \frac{\exp \alpha_i S_i}{\exp \alpha_i S_i + \sum_{k \neq i} p(\theta_j)\alpha_j S_j} & i = j \\ \frac{\exp p(\theta_j)\alpha_j S_j}{\exp \alpha_i S_i + \sum_{k \neq i} p(\theta_k)\alpha_k S_k} & i \neq j \end{cases}$$

2. Workers – Choosing a sector

- Using GEV trick, express worker's value functions as

$$W_i = w_i + \beta \left(\delta_i \cdot U + (1 - \delta_i) \cdot EW_i\right)$$
$$U = b + \beta \cdot EW$$

where

$$EW = \mathbb{E}\left[\max_{k \in \mathcal{I}} \left\{\mathcal{W}_k - \varphi_k^U + \epsilon_i\right\}\right] = \sum_{k=1}^{n} s_k^U \cdot \left(\mathcal{W}_k - \varphi_k^U\right)$$

$$EW_i = \mathbb{E}\left[\max\left\{W_i + \epsilon_i, \max_{k \neq i}\left\{\mathcal{W}_k - \varphi_{ik}^E + \epsilon_k\right\}\right\}\right] = s_{ii}^E \cdot W_i + \sum_{k \neq i}^{n} s_{ij}^E \cdot \left(\mathcal{W}_j - \varphi_{ik}^E\right)$$

$$\mathcal{W}_i = p\left(\theta_i\right) \cdot W_i + \left(1 - p\left(\theta_i\right)\right) \cdot U$$

3. Wage determination – Nash bargaining

- Wages are such so that workers receive a share α_i of total surplus.
- Characterization of the surplus function

$$S_i(\mathbf{z}) = (\pi_i(\mathbf{z}) + \kappa_i - b) - \beta\mathbb{E}_{\mathbf{z}}\left\{(1-\delta_i)\sum_{j\neq i} s_{ij}^E(\mathbf{z}')\varphi_{ij}^E - \sum_{j=1}^{n} s_j^U(\mathbf{z}')\varphi_j^U\right\}$$
$$+ \beta\mathbb{E}_{\mathbf{z}}\left\{\left(1-\delta_i^F(\mathbf{z}') - s_i^U(\mathbf{z}')p(\theta_i(\mathbf{z}'))\alpha_i - q(\theta_i(\mathbf{z}'))(1-\alpha_i)\right)S_i(\mathbf{z}')\right\}$$
$$+ \beta\mathbb{E}_{\mathbf{z}}\left\{\sum_{j\neq i}\left[\left((1-\delta_i)s_{ij}^E(\mathbf{z}') - s_j^U(\mathbf{z}')\right)p(\theta_j(\mathbf{z}'))\alpha_j S_j(\mathbf{z}')\right]\right\}$$

- Standard term in search models
- Moving costs $(\equiv \Phi(\mathbf{z}))$
- Additional term if worker moves to another sector

Laws of Motion

- Period t begins with a mass of unemployed workers u_{t-1}, and employed workers $\mu_{t-1} \equiv \sum_{i=1}^{n} \mu_{i,t-1}$.
- Mass of *exogenous* job destruction $\Delta_t \equiv \sum_{i=1}^{n} \delta_i \mu_{i,t-1}$
- UE transitions: $\phi^U u_{t-1}$, where $\phi^U = \sum_{j=1}^{n} s_j^U p(\theta_j)$
- J2J transitions (from sector i): $\phi_i^E \mu_{i,t-1}$, where $\phi_i^E = \sum_{j \neq i} p(\theta_j) s_{ij}^E$
- Stayers in sector i: $(1 - \delta_i^F) \mu_{i,t-1}, \; \delta_i^F = (1 - \delta_i) s_{ii}^E$

Laws of Motion (con't)

- Law of motion for unemployment,

$$u_t = \left(1 - \phi^U\right) u_{t-1} + \Delta_t + \Lambda_t$$

where $\Lambda_t = \sum_{j=1}^{n} \left(1 - \phi_j^E - s_{ii}^E\right) (1 - \delta_i)\, \mu_{j,t}$ is the measure of EU transitions

- Law of motion for sectoral employment

$$\mu_{i,t} = \underbrace{\left(1 - \delta_i^F\right) \mu_{i,t-1}}_{\text{stayers}} + p\left(\theta_i\right) \left[\underbrace{s_i^U u_{t-1}}_{\text{UE}} + \underbrace{\sum_{j \neq i} s_{ji}^E \left(1 - \delta_i\right) \mu_{j,t-1}}_{\text{intersectoral EE}} \right]$$

- Mass of jobseekers in sector i $(\equiv \psi_{i,t})$

Stationary Distributions

- In a stationary environment, we have that

$$\phi^U u = \Delta + \Lambda, \quad \delta_i^F \mu_i = p(\theta_i)\,\psi_i$$

$$\Rightarrow u = \left(\phi^U\right)^{-1} \sum_{j=1}^{n} \left(\delta_i^F - \phi_i^E \left(1 - \delta_j\right)\right) \mu_j$$

$$\mu_i = \frac{p(\theta_i)}{\delta_i^F} \left[s_i^U u + \sum_{j \neq i} s_{ji}^E \left(1 - \delta_j\right) \mu_j \right]$$

- Recall from standard search model: $\delta u^* = \lambda \left(1 - u^*\right)$
- Mass of workers is normalized to 1 $\Rightarrow u_t + \mu_t = 1$

Aggregate demand

- Full risk–sharing + no savings decision
- Family consume aggregate consumption good C
- Representative final good producer

$$C = \left[\sum_{i=1}^{n} a_i^{1/\sigma} c_i^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \quad \Rightarrow c_i^* = a_i p_i^{-\sigma} Y$$

- Price index (normalized to 1)

$$P = \left[\sum_{i=1}^{n} a_i p_i^{1-\sigma} \right]^{1/(1-\sigma)}$$

Aggregate demand

- Market clearing

$$\mu_i y_i = c_i + \sum_{j=1}^{n} x_{ji} \quad i = 1, \ldots, n$$

$$u + \mu = 1$$

- Resource constraint

$$\Pi = \sum_{i=1}^{n} \mu_i \pi_i = C + \sum_{i=1}^{n} \theta_i \psi_i \kappa_i$$

since ψ_i is the mass of jobseekers in sector i

Calibration

- Some parameters can be calibrated directly from the data: Ω, β, $\mathbf{a}$
- Remaining parameters can be estimated using simulation–based methods (SMM, AMM!)

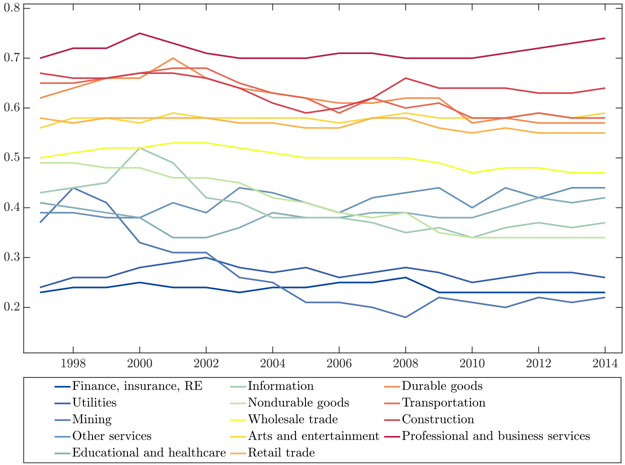
Parameter	Target
η	Sectoral Labor shares
α	Wage growth
κ	Vacancies, firm entry
δ	Sectoral JD rate
Φ, M	EE, UE transitions

Road Ahead

1. Compare *network* + *search* economy with...
   - No production networks: $\eta = 0$
   - No search frictions: $\kappa = 0, p(\theta), q(\theta) = 1$
   - Different network structures: Ω
2. At this point, no worker heterogeneity.
   - Introduce occupation networks!

Thanks!

Sectoral labor shares – Data



Sectoral Labor shares – Model

- Free entry condition

$$J_i(\mathbf{z}) = \pi_i(\mathbf{z}) - w_i + \beta\left(1 - \delta_i^F\right) J_i(\mathbf{z})$$
$$0 = -\kappa_i + p(\theta_i) J_i(\mathbf{z})$$

gives

$$w_i = \pi_i(\mathbf{z}) - \left[\frac{1 - \beta\left(1 - \delta_i^F\right)}{p(\theta_i)}\right]\kappa_i$$
$$L_i^S \simeq (1 - \eta_i) - \frac{\delta_i^F}{p(\theta_i)} \frac{\kappa_i}{p_i y_i(\mathbf{z})}$$